Chapter 5 Review

Pgs 315 # 9, 11, 13-33 odd, 38, 39, 41, 45, 49, 51

(9) a) true

b) true: $\int_{-2}^{5} f(x) = \int_{-2}^{2} f(x) + \int_{2}^{5} f(x)$

$= 4 + 3 = 7$

$\int_{-2}^{5} f(x) + g(x) = 7 + 2 = 9$

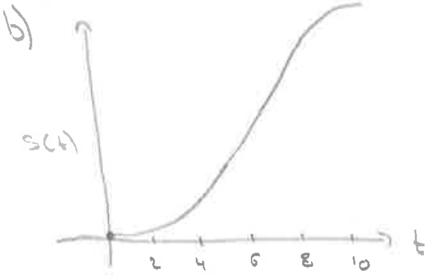
c) False: $\int_{-2}^{5} f(x) = 7$, $\int_{-2}^{5} g(x) = 2$

(11) Trap rule, $n = 5$, $h = 2$

a) $A = \frac{1}{2} \cdot 2[0 + 2(1.2) + 2(3.4) + 2(4.75) + 2(3.5) + 0]$

≈ 25.7

b)



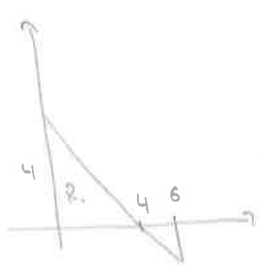
s(t) increasing, v(t) always > 0

(12) a) $\int_{0}^{10} x^3\, dx$

b) $\int_{0}^{10} x \sin x\, dx$

c) $\int_{0}^{10} x(3x-2)^2\, dx$

(13)



$A = R_1 + R_2$

$= \frac{1}{2}(4)^2 + \frac{1}{2}(2)^2$

$= 8 + 2$

$= \underline{10}$

$A = \int_{0}^{4} (4-x)\,dx - \int_{4}^{6} (4-x)\,dx$

(15) $\int_{-2}^{2} 5\, dx = 5x\Big]_{-2}^{2} = 10 - -10$

$= \underline{20}$

(17) $\int_{0}^{\pi/4} \cos x\, dx = \sin x\Big]_{0}^{\pi/4} = \frac{\sqrt{2}}{2} - 0$

$= \underline{\frac{\sqrt{2}}{2}}$

(19) $\int_{0}^{1} (8s^3 - 12s^2 + 5)\, ds$

$= \Big[2s^4 - 4s^3 + 5s\Big]_{0}^{1} = 3 - 0$

$= \underline{3}$

(21) $\int_{1}^{27} y^{-4/3}\, dy = -3y^{-\frac{1}{3}}\Big]_{1}^{27}$

$= \frac{-3}{3} - -3$

$= -1 + 3 = \underline{2}$

(23) $\int_{0}^{\pi/3} \sec^2\theta\, d\theta = \tan\theta\Big]_{0}^{\pi/3}$

$= \sqrt{3} - 0 = \underline{\sqrt{3}}$

(25) $\int_{0}^{1} \frac{36}{(2x+1)^3}\, dx = 36\int_{0}^{1} \frac{1}{(2x+1)^3}\, dx$

$= 36\int_{0}^{1} (2x+1)^{-3}\, dx$

$= 36\Big[-\frac{1}{4}(2x+1)^{-2}\Big]_{0}^{1}$

$= 36\Big[-\frac{1}{4}\left(\frac{1}{9}\right) + \frac{1}{4}\Big]$

$= -1 + 9$

$= \underline{8}$

(27) $\int_{-\pi/3}^{0} \sec x \tan x \, dx = \sec x \Big]_{-\pi/3}^{0}$

$= 1 - 2 = -1$

(29) $\int_0^2 \frac{2}{y+1} \, dy = 2\int_0^2 \frac{1}{y+1} \, dy$

$= 2\left[\ln(y+1)\right]_0^2$

$= 2(\ln 3 - \ln 1)$

$= 2 \ln 3$

(31) $\int_{-4}^{8} |x| \, dx$

$= \frac{1}{2}(4)^2 + \frac{1}{2}(8)^2$

$= 8 + 32 = 40$

(33) each interval is 1 day = 24 hrs

a) upper (like RRAM)

$24(.020 + .021 + .023 + .025 + .028 + .031 + .035)$

$= 4.392$ L

Lower (LRAM)

$24(.019 + .020 + .021 + .023 + .025 + .028 + .031)$

$= 4.008$ L

b) $h = \frac{24 \cdot 7}{7} = 24$

$A = \frac{24}{2}\big[.019 + 2(.020) + 2(.021) + 2(.023) + 2(.025)$

$+ 2(.028) + 2(.031) + .035\big]$

$= 4.2$ L

(37) $0 \le \int_0^1 \sqrt{1+\sin^2 x} \, dx \le \sqrt{2}$

$f(x) = \sqrt{1+\sin^2 x}$

max $f(x) = \sqrt{2}$ since max $\sin^2 x = 1$

min $f(x) = 1$ since min $\sin^2 x = 0$

$(1-0) \min f(x) \le \int_0^1 \sqrt{1+\sin^2 x} \, dx \le (1-0) \max f$

$0 < 1 \le \int_0^1 \sqrt{1+\sin^2 x} \, dx \le \sqrt{2}$ ✓

(38) $av(f) = \frac{1}{4}\int_0^4 \sqrt{x} \, dx$

$= \frac{1}{4}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4$

$= \frac{1}{4}\left(\frac{16}{3} - 0\right) = \frac{4}{3}$

b) $av\, f = \frac{1}{a}\int_0^a a\sqrt{x} \, dx$

$= \int_0^a \sqrt{x} \, dx$

$= \frac{2}{3}x^{\frac{3}{2}}\Big]_0^a = \frac{2}{3}a^{3/2}$

(39) $y = \int_2^x \sqrt{2 + \cos^3 t} \, dt$

$\frac{dy}{dx} = \sqrt{2 + \cos^3 x}$ (FTC)

(41) $y = \int_x^1 \frac{6}{3+t^4} \, dt$

$\frac{dy}{dx} = -\frac{6}{3+x^4}$ (FTC)

(45) $\int_0^x (t^3 - 2t + 3)\,dt = 4$

$\frac{1}{4}t^4 - t^2 + 3t\Big]_0^x = 4$

$\frac{1}{4}x^4 - x^2 + 3x = 4$

$x^4 - 4x^2 + 12x - 16 = 0$

$x \approx -3.091$, $x \approx 1.631$

(49) $y = x^2 + \int_1^x \frac{1}{t}\,dt + 1$

i) $y' = 2x + \frac{1}{x}$

$= 2x + x^{-1}$

$y'' = 2 - \frac{1}{x^2}$

ii) $x = 1$, $y = 1^2 + \int_1^1 \frac{1}{t}\,dt + 1$

$= 1 + 0 + 1$

$= \underline{2}$

$y' = 2(1) + \frac{1}{1} = \underline{3}$

(51) Each interval is $\frac{5}{60}$ $h = \frac{1}{12}$

Total fuel $\approx \frac{1}{2} \cdot \frac{1}{12}[2.5 + 2(2.4) + 2(2.3) + \ldots + 2(2.4) + 2.3]$

$\approx 2.41\overline{6}$ gal/h

b) $\left(60 \frac{mi}{h}\right)\left(\frac{1}{2.416} \frac{h}{gal}\right) = 24.828 \ mi/gal$

Ps 315 Quick Quiz for AP Prep

① $\int_1^7 f(x)\,dx \approx T = \frac{1}{2}\cdot 3(10+30) + \frac{1}{2}\cdot 2(30+40) + \frac{1}{2}\cdot 1(40+20)$

$= \underline{160} \quad - C$

② $f(x) = \sin^3 x, \quad \int_1^8 f(x)\,dx = F(8) - F(1)$

$\int_1^8 \sin^3 x = F(8) - 0$

$\underline{.6322 = F(8)} \quad - D$

③ $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\,dt$

$f'(x) = e^{(x^2-3x)^2}\cdot(2x-3)$ $\qquad$ $\underline{\text{min at } x = \frac{3}{2} \quad - C}$

NLA

$e^{(x^2-3x)^2}$	$+$		$+$
$2x-3$	$-$		$+$
	$\ominus$ $\searrow$	$\frac{3}{2}$	$\oplus$ $\nearrow$

④ $F(x) = \int_0^x \sin(t^2)\,dt, \quad 0 \le x \le 3$

a) $F(2) = \int_0^2 \sin(t^2)\,dt \approx T = \frac{1}{2}\cdot\frac{1}{2}\left(\sin(0) + 2\sin\left(\left(\tfrac{1}{2}\right)^2\right) + 2\sin(1^2) + 2\sin\left(\left(\tfrac{3}{2}\right)^2\right) + \sin(2^2)\right)$

$= \underline{.7413}$

b) $F'(x) = \sin(x^2) > 0$

Look at graph of $F'(x)$ on $0 \le x \le 3$

increasing on $(0, \sqrt{\pi})$ and $(\sqrt{2\pi}, 3)$

c) $\frac{F(3) - F(0)}{3} = \frac{1}{3}\int_0^3 \sin(t^2)\,dt = k$

$\int_0^3 \sin(t^2)\,dt = 3k$

Pg 319 AP Exam Prep

(58)

a) $T = \int_0^{24} R(t)\,dt$

$T = \frac{1}{2} \cdot 4\,(9.6 + 2(10.3) + 2(10.9) + \ldots + 9.6)$

$= 253.2$ gallons of water came out of the pipe in a 24 hr period.

b) Yes; $\dfrac{R(24) - R(0)}{24 - 0} = 0$. MVT says there is a point c in $[0, 24]$ s.t. $R'(c) = 0$

c) $\text{Ave}(Q) = \frac{1}{24}\int_0^{24} Q(t)\,dt$

$= 10.58$ gal/hr

(60) a) $\int_1^4 f(t)\,dt = 2$

$\int_1^{-2} f(t)\,dt = -\frac{9}{2}$

area

b) $g'(2) = f(2) = 1$

c) min at $x = -2$

$g(-2) = -\frac{9}{2}$

d) at $x = 1$

$g''(x) = f'(x)$

since $f'(x)$ has a sign change at $x = 1$.

⇒

(59) $f'(x) = ax^2 + bx$ $\quad f''(x) = 2ax + b = 6$

$-6 = a + b$ $\quad 2a + b = 6$

$b = 6 - 2a$

$-6 = a + 6 - 2a$

$a = 12$

$b = -18$

$f'(x) = 12x^2 - 18x$

$f(x) = 4x^3 - 9x^2 + C$

$\int_1^2 \left[4x^3 - 9x^2 + c\right] dx = 14$

$\left. x^4 - 3x^3 + cx \right]_1^2 = 14$

$(16 - 24 + 2c) - (1 - 3 + c) = 14$

$c = 20$

$\boxed{f(x) = 4x^3 - 9x^2 + 20}$

Pg 312

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a) $g(1) = \int_1^1 f(t)\,dt = 0$

b) $g(3) = \int_1^3 f(t)\,dt = -1$ (area)

c) $g(-1) = \int_1^{-1} f(t)\,dt = -\pi$ (area)

d) max at $x = 1$ (greatest area)

e) $g(-1) = -\pi$ point

$g'(-1) = f(-1) = 2$ slope

$y + \pi = 2(x+1)$

$y = 2x + 2 - \pi$

f) $g''(x) = f'(x)$ look where f changes from increasing to decreasing or decreasing to increasing:

$x = -1, \quad x = 2$

g) $[-2\pi, 0]$